A hydro-bio-mechanical model for settlement and other behaviour in landfilled waste

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Abstract: A conceptual framework for the analysis of settlement and other processes in landfilled waste in relation to the coupled hydraulic, biodegradation and mechanical behaviour is presented. The hydraulic model is an unsaturated flow model. The biodegradation model is a two-stage anaerobic digestion model modified to account for enzymatic hydrolysis of solid degradable fraction in relation to moisture content, product inhibition, cellulose digestibility and microbial controls. The mechanical model includes an innovative interpretation of the mechanical consequences of decomposition combined with established formulations of load and creep-induced effects to predict settlement under load and over time. A finite element implementation enables simulation of both the filling phase and leachate control practices. Finally, the formulation is tested by comparison with a large-scale laboratory waste compression test. This paper is rewritten on the basis of McDougall (2007).

1   Introduction & conceptual framework

Total settlement in untreated landfilled municipal solid waste (MSW) has been estimated to range between 25% and 50% of initial fill height (Wall & Zeiss, 1995), of which more than half may be attributable to long term or secondary settlement. Settlement is a major constraint on the successful management of a landfill site because:

- excessive settlement can lead to failure of the capping layer, leachate or gas collection systems,
- re-use of closed sites can only be done with knowledge of the potential settlement, and
- accurate prediction of settlement enables maximum use of available void space.

An understanding of, and ability to analyse, landfill settlement is complicated by the coincidence of load, creep and biodegradation-induced effects. Constitutive frameworks for load and creep effects are well developed but the presence and impact of a solid degradable fraction are not properly accounted for in conventional geomechanical models. Typically, a temporal classification is used. Load-induced settlement is regarded as a immediate settlement process and interpreted by a constrained modulus. Creep and biodegradation-induced settlement are combined into a single secondary settlement process that is usually handled by a time-dependent function. Such functions may be expedient for secondary settlement where biodegradation is inhibited but they do not easily accommodate the acceleration in settlement often observed in landfills after closure (Bjarngard & Edgers, 1990). The problem is due to the fact that biodegradation is controlled by environmental factors such as the amount and nature of the organic fraction, moisture content (as a pre-requisite for microbial activity, a reactant and vector for microbial diffusion), moisture quality, microbial populations and pH. None of these factors are constrained to evolve in a simple time-dependent manner. In the absence of a more fundamental interpretation of biodegradation, geomechanical analysis is forced to take a highly empirical and observational approach to landfill settlement.

A framework for the integrated analysis of the hydraulic, biodegradation and mechanical behaviour of landfilled municipal solid waste (MSW) or other degradable soils, referred to as the HBM model is here presented. The HBM framework comprises three main system models describing hydraulic, biodegradation and mechanical behaviour respectively. The system models are connected by link routines as shown in Fig.1.

As system models, the hydraulic model and certain elements of the mechanical model are the basis of well-established design tools. For example, the hydraulic model is a two-dimensional formulation of Richards' equation (Richards, 1936); the mechanical model is a plane-strain, formulation of Modified Cam Clay (Roscoe & Burland, 1968) with vertical creep handled by the 'equivalent time' method of Yin & Graham (1989). The biodegradation model is anaerobic digestion model but includes a number of innovations.
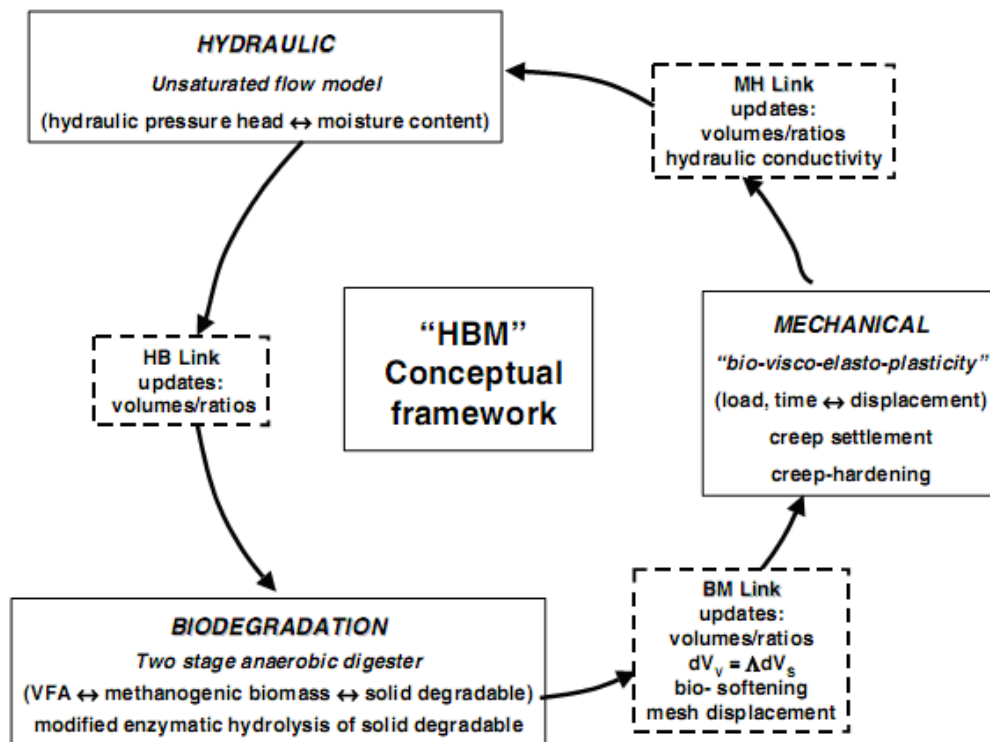
Figure 1. Schematic representation of the HBM conceptual framework.

By using relatively simple system models, input parameter requirements are kept relatively light. However, at the heart of the HBM framework is the communication between system models. Field variables of one system define important parameters or have a direct influence upon one of the other systems. For example, moisture content, which is a field variable in the hydraulic model is a controlling parameter in the biodegradation model. Mass loss has a direct impact on phase composition as solid volume is lost and particle rearrangement ensues. Changes in phase composition affect hydraulic conductivity and, in the case of mass loss, may result in hydraulic conductivities increasing with decomposition. The link routines contain procedures to enable the most recent field variable values to be used by all system models. In some cases this is simply a matter of updating parameter values; in other cases, such as the determination of solids loss and its interpretation within the mechanical model, further analytical procedures are invoked. These further analytical procedures and system interdependencies are a challenging aspect of model development. If they can be sensibly understood, then hitherto disparate behaviours can be analysed in a more meaningful and coherent context.

Whilst certain limitations to the framework exist, and will be highlighted in relevant sections below, it is important to realise that the aim of the HBM model, and of this paper, is to present a more fundamental and synergistic interpretation of hydraulic, biodegradation and mechanical behaviours in landfilled waste.

2   Hydraulic model

The hydraulic model is an unsaturated flow model in which the main system variables are hydraulic pressure head and moisture content. It is a two-dimensional formulation of Richards' equation (Richards, 1931),

$$\frac{\partial}{\partial x} k_x(\theta) \frac{\partial \psi}{\partial x} + \frac{\partial}{\partial z} k_z(\theta) \frac{\partial \psi}{\partial z} + \frac{\partial k_z(\theta)}{\partial z} = C_w \frac{\partial \psi}{\partial t} \qquad (1)$$

where ψ is the hydraulic pressure head [m], k denotes the hydraulic conductivity, θ is volumetric moisture content, t is time, C_w $(= \partial\theta/\partial\psi)$ is the specific water capacity, and x & z are space coordinates.

2.1 Parameter requirements

Parameter requirement is summarised in Table 1; justification for the selection of hydraulic parameter values is given in the relevant sections.

Table 1: Hydraulic model parameters and values for MSW

	Input parameter	*Dimensions*	*Waste*
H1	Van Genuchten α		1.4
H2	Van Genuchten n		1.6
H3	Residual moisture content (w/w)		0.25
H4	Specific storage		0
H5	Saturated hydraulic conductivity	$m.s^{-1}$	5×10^{-5}
H6	Anisotropy w.r.t. hydraulic conductivity, ratio horizontal/vertical		1

2.2 Moisture retention

It is a fundamental feature of an unsaturated soil that the amount of moisture retained is related to the (negative) porewater pressure or suction and evident from Eq (1) that a suction–moisture content relationship is required for the solution of unsaturated flow problems. In the HBM model, van Genuchten's (1980) formula is used to express this relationship, i.e.

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{\left[1 + (\alpha|\psi|)^n\right]^m}$$

(2)

where the absolute value of ψ is used, θ_r and θ_s are the residual and saturated volumetric moisture contents respectively, α, n and m are model parameters. Figure 2 shows a moisture retention curve, effectively the equilibrium moisture profile, with the meaning of the van Genuchten parameters highlighted.
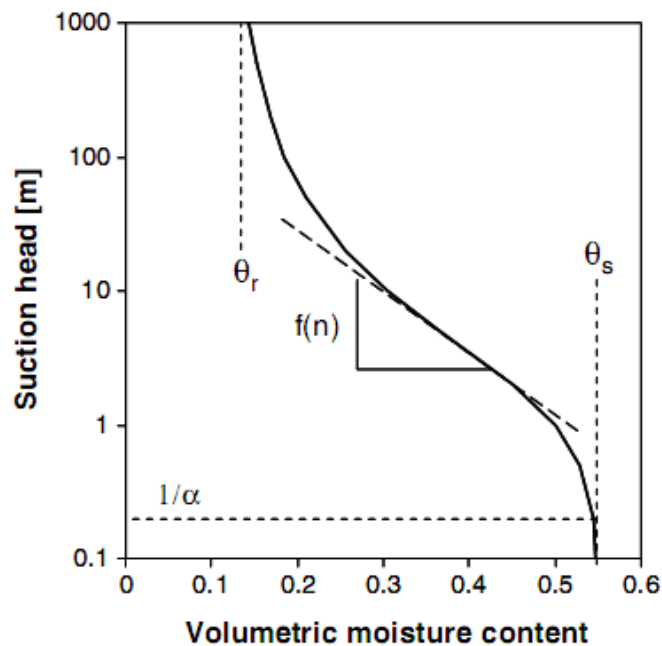

Figure 2. Moisture retention curve with meaning of van Genuchten parameters.

Volumetric moisture content is a physical soil property. The residual moisture content is the moisture content below which little change in moisture content occurs, regardless of the applied pressure head. Neutron probe data obtained by Yuen (1999) at the Lyndhurst Sanitary Landfill in Victoria, Australia show the upper elevations (0-4m) of a 12 m deep landfill to have reached a relatively uniform gravimetric moisture content of about 25%. It is this value that has been assumed for the gravimetric residual moisture content and from which the volumetric form is calculated. The saturated moisture content is equivalent to the waste porosity, and is initially calculated from unit weight data and thereafter updated with compression and/or decomposition.

Values for parameters α & n were obtained from laboratory tests (Kazimoglu et al, 2005), the retention curve from which is shown in Fig. 3. Parameter m has no physical interpretation and is often fixed as a function of n,
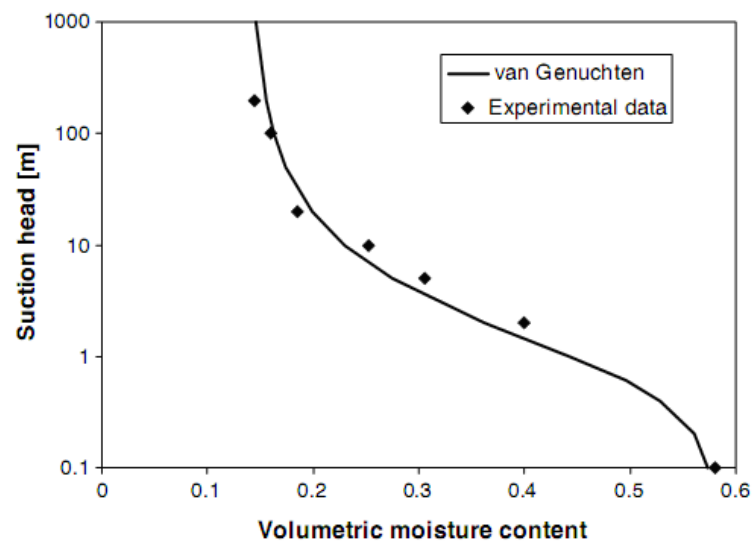
$$m = 1 - \frac{1}{n} \tag{3}$$

Figure 3. Moisture retention in compacted waste sample obtained using modified pressure plate apparatus, from Kazimoglu et al. [11] and fitted van Genuchten curve where α = 1.4 and n = 1.6.

2.3 Hydraulic conductivity

Moisture flow is assumed to comply with Darcy's Law. In unsaturated zones the coefficient of permeability or hydraulic conductivity $k(\theta)$, is dependent on the amount of moisture,

$$k(\theta) = k_{sat} k_r(\theta) \tag{4}$$

where k_{sat} is the saturated hydraulic conductivity and k_r is the relative permeability. The relative permeability, which is a function of the moisture saturation, is defined using van Genuchten's (1980) function,

$$k_r(\theta) = \sqrt{\theta_e} \left[1 - \left(1 - \theta_e^{1/m} \right)^m \right]^2 \tag{5}$$

where θ_e is the effective moisture content,

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{6}$$

2.3.1   Phase-dependent saturated hydraulic conductivity

It is known that the saturated hydraulic conductivities of a sample of waste can vary by three or more orders of magnitude when compressed over the range of stresses exerted within 20m depth of waste (Powrie et al., 1998). In response, the HBM model allows for the saturated hydraulic conductivity to be controlled by the void phase volume of each element. A relationship between saturated hydraulic conductivity and the void phase volume as a ratio of the solid inert phase volume is used. The influence of decomposition on hydraulic conductivity, through its impact on void volume, can be realised as follows.

From data obtained by Beaven (2000) on the saturated hydraulic conductivity of household wastes in a large (2m diameter x 2m high) compression cell shown in Fig.4, it is evident that the relationship between dry unit weight and saturated hydraulic conductivity can be approximated by a function of the form,

$$k_{sat} = B \exp(c.\gamma_d) \tag{7}$$

where γ_d is the dry unit weight, B and c are fitting parameters. Approximate values of B = 0.15 and c = −2.0 can be seen for DM3, a MSW, shown in Fig.4. However, in a degrading soil comprising inert and degradable solid phases, which have different phase densities (or specific gravities), the overall dry unit weight does not necessarily correspond to a unique volumetric state.
A more useful controlling variable would be the void to inert phase ratio, e_i, given by,

$$e_i = \frac{V_V}{V_{SI}} \tag{8}$$

where V_V is the void volume and V_{SI} is the solid inert phase volume (McDougall & Pyrah, 2004). By assuming an inert/degradable waste composition and corresponding unit weights, eq (8) can be expanded to define e_i in terms of overall dry density, i.e.

$$e_i = \frac{G_{SI}.\gamma_w}{\gamma_d(1-\omega)} - \frac{G_{SI}.\gamma_w.\omega}{G_{SD}.\gamma_w.(1-\omega)} - 1 \tag{9}$$

where G_{SI} is the specific gravity of the inert phase component, G_{SD} is the specific gravity of the degradable phase component, γ_w is the unit weight of water and ω is the mass fraction of solid degradable matter as a proportion of total solid mass.

Data from Figure 4 can then be re-interpreted as a function of e_i. Figure 5 shows the DM3 conductivity data so calculated, assuming the solid degradable dry weight fraction of sample DM3 is 0.54, the specific phase weights of the inert phase $G_{SI}.\gamma_w = 17$ kN/m³ and degradable phase $G_{SD}.\gamma_w = 7.3$ kN/m³. The fitted function is logarithmic in form,

$$k_{sat} = 1.0e^{-8} x(e_i)^{4.9} \tag{10}$$

Equation (10) is currently hard-coded into the HBM model and can be selected in preference to a fixed input saturated hydraulic conductivity. One should be aware, however, that preferential flow pathways are common in waste so bulk saturated hydraulic conductivity might misrepresent field conditions.
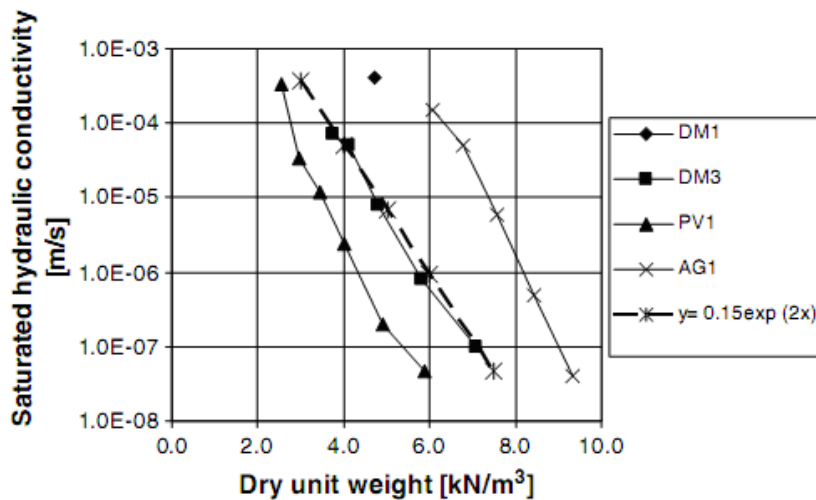

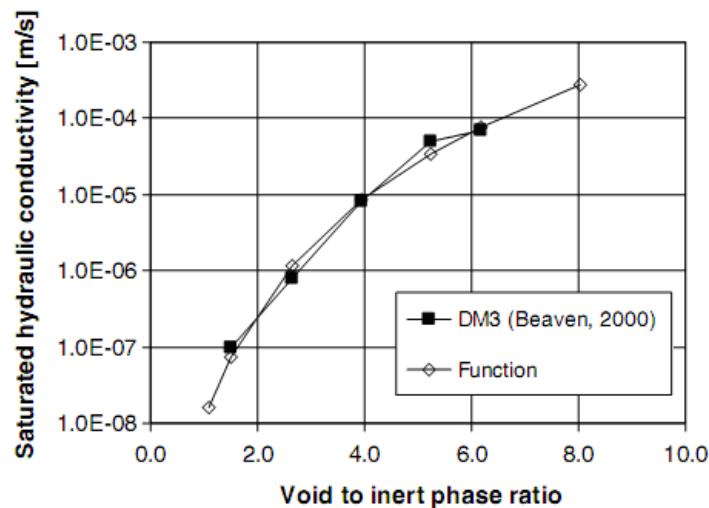Figure 4. Variation of saturated hydraulic conductivity with dry unit weight, from data published by Beaven [2].

Figure 5. Waste saturated hydraulic conductivity data (DM3 from [2]) presented as a function of void to inert phase ratio.

3   Biodegradation model

The biodegradation model describes a two-stage anaerobic digester in which token volatile fatty acid (VFA) and methanogenic biomass (MB) concentrations are the main field variables. Solid degradable fraction (SDF) depletion is calculated for each time step based on the contemporaneous VFA and MB concentrations and moisture content.

3.1   Parameter requirements

Parameter requirement is given in Table 2; justification for the selection of biodegradation parameter values is given in the relevant sections.

Table 2. Biodegradation model parameters & values for MSW

|	*Input parameter*	*Dimensions*	*Waste*
B1	Maximum hydrolysis rate	$g.m^{-3}_{(aq)}.day^{-1}$	2500
B2	Product inhibition	$m^3.g^{-1}$	$2x10^{-4}$
B2	Digestibility		0.7
B3	Half rate	$g.m^{-3}$	4000
B5	Methanogen growth	day^{-1}	0.02
B6	Methanogen death	day^{-1}	0.002
B7	Yield coefficient		0.08
B8	Diffusion coefficient	$m^2.day^{-1}$	0.05
I1	Initial solid degradable fraction		0.4
I2	Initial VFA concentration	$g.m^{-3}$	300
I3	Initial methanogenic biomass	$g.m^{-3}$	250

3.2   Mineralisation - Metabolic pathway

The mineralisation of organic matter is a microbially-mediated process, dependent on the nature of the substrate and environmental conditions such as availability of moisture. Because cellulolytic matter accounts for 91% of the methane potential of waste refuse (Barlaz et al, 1989), mineralization in landfill is usually defined by a metabolic pathway of the kind commonly associated with the anaerobic digestion of cellulose. The process is idealised by three main steps:

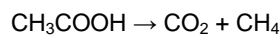1. Enzymatic hydrolysis of solid cellulose ($C_6H_{10}O_5$) to glucose ($C_6H_{12}O_6$).

$$C_6H_{10}O_5 + H_2O \rightarrow C_6H_{12}O_6$$
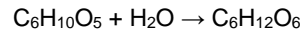
2. Fermentation of glucose to acetic acid (CH_3COOH), a token VFA.

$$C_6H_{12}O_6 + 4H_2O \rightarrow CH_3COOH + 8H_2 + 4CO_2$$
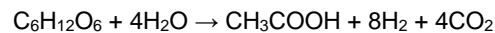
3. Methanogenesis,

i)   by acetate cleavage

$$CH_3COOH \rightarrow CO_2 + CH_4$$

ii)   by hydrogen-scavenging bacteria.

$$8H_2 + 2CO_2 \rightarrow 2CH_4 + 4H_2O$$
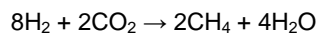
Given the relative vigour of the fermentative processes of acidogenesis and acetogenesis (step 2) by comparison to methanogenesis (step 3), and assuming hydrogen-scavenging methanogens consume all gaseous intermediates (step 3(ii)), the stoichiometry and mass balance of the three main steps can be simplified as follows:

$$C_6H_{10}O_5 + H_2O \rightarrow CH_3COOH + 8H_2 + 4CO_2 \rightarrow 3CO_2 + 3CH_4$$

162g   18g   60g   132g   48g
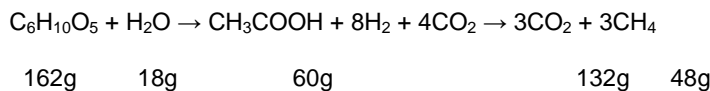
In mass terms, the hydrolysis of 1 mole of cellulose (MW = 162g) produces 3 moles of carbon dioxide (= 132g) and 3 moles of methane (= 48g).

3.3   Mineralisation - Kinetics of enzymatic hydrolysis

Unlike more diffuse aqueous substrates such as sewage sludge, waste refuse is predominantly a solid structured material in which the rate and progress of decomposition are constrained by physical factors. In the HBM model, modifications have been made to reflect more realistically the physical factors influencing the solid to aqueous phase transfer. A modified enzymatic hydrolysis function accounts for the influence of the changing digestibility of the degradable fraction, product inhibition and moisture content on hydrolysis

3.3.1   Maximum rate of hydrolysis

This is the maximum or initial rate of hydrolysis of solid organic matter occurring under the most favourable substrate structure and interaction conditions. The maximum hydrolysis rate determines the accumulation of VFA, principally acetic acid, in the aqueous phase. It has units of mass per unit volume aqueous phase per unit time, i.e. $g_{VFA}.m^{-3}_{aqueous}.day^{-1}$. Estimates of the maximum hydrolysis rate can be made from VFA growth vs. time plots, e.g. from Barlaz et al (1989) wherein a peak VFA growth rate of about 1800 $mg_{VFA}.L^{-1}_{aqueous}.day^{-1}$ can be noted. Jones & Grainger (1983) indicate a maximum accumulation of about 3000 $g_{VFA}.m^{-3}_{aqueous}.day^{-1}$.

Alternatively, the loss of solids may be considered. Cecchi et al (1988) and Wang & Banks (2000) indicate maximum volatile solid (VS) reduction rates in the range 4000 to 5000 $g_{totalVS}.m^{-3}_{aqueous}.day^{-1}$. It is important to note here that the difference in molecular weights between cellulose or glucose and VFA, means that the solids reduction and VFA accumulation data above are more consistent than they appear. This matter is discussed in more detail in the section on substrate depletion, below. That said, work by Lee & Fan (1983) and more recently by Rodriguez (2005) on the in-vitro decomposition of small samples of cellulose, indicate that laboratory determined values of the maximum hydrolysis of shredded waste are higher. This is an issue that deserves further investigation but in the meantime, a maximum hydrolysis rate value of 2500 $mg_{VFA}.L^{-1}_{aqueous}.day^{-1}$ has been adopted.

3.3.2 Digestibility

The presence of highly degradable organic matter and/or the initial colonisation and enzymatic attack of exposed waste surfaces mean that initial hydrolysis rates are high. Remaining organic matter, having become less accessible to, or shielded from, enzymatic attack, or with an increased crystallinity, becomes less digestible and is hydrolysed at slower rates. Lee & Fan (1982) suggested that a lumped parameter, referred to as the relative digestibility, be used to reflect the combined effects of changes in accessible surface area and crystallinity. They found relative digestibility, ϕ, to be related to the extent of substrate conversion by a single parameter, n, the structural transformation parameter, i.e.

$$\phi = 1 - \left[\frac{S_o - S}{S_o} \right]^n \tag{11}$$

where S is the solid degradable fraction remaining and S_o is the initial solid degradable fraction. Solid degradable fraction is reported as a mass per unit total volume, i.e. in kg/m^3. Lee & Fan reported a value of 0.36 for the structural transformation parameter but indicated that the value is probably strongly dependent on the structural features of the cellulose. In their tests, "Solka-Floc", a commercially available delignified cellulose was used but lignin, a substance which is resistant to enzymatic hydrolysis and can shield cellulose, comprises up to 15% by dry weight of the organic fraction of waste refuse (Bookter & Ham, 1982). Calculations performed on data presented by Wald et al. (1984) for rice straw, a lignified cellulose, reveal a higher value (0.7) for the structural transformation parameter.

3.3.3 Product inhibition

A product inhibition term P, based on VFA concentration, allows for acid accumulation and associated 'souring' of a site,

$$P = \exp(-k_{VFA}(c)) \tag{12}$$

where k_{VFA} is the product inhibition factor and c is the concentration of volatile fatty acids [units = $g_{VFA}.m^{-3}_{aqueous}$]. Values for k_{VFA} are chosen to ensure that maximum VFA concentrations correspond to those reported in sites or other installations that are known to have 'soured' or 'stuck'. Typically, a value of 2×10^{-4} m^3/g results in peak VFA values of about 16,000 g/m^3.

3.3.4 Moisture content

One of the most important influences on the biodegradation of landfilled waste, and one that can be controlled most easily during the life of a landfill, is moisture. Moisture content and flow, act as a means by which chemical substances and microbes penetrate the waste mass and as a pre-requisite for microbial function.

3.3.5 Modified enzymatic hydrolysis function

Combining the maximum hydrolysis rate b, limiting factors ϕ (Eq.11) and P (Eq.12) and an effective moisture content term (Eq. 6) gives an equation describing the enzymatic hydrolysis of waste refuse under a range of moisture contents,

$$r_g = \theta_E \, b \left[1 - \left[\frac{S_O - S}{S_O} \right]^n \right] \exp(-k_{VFA}(c)) \tag{13}$$

where r_g denotes the rate of VFA accumulation in $g_{VFA}.m^{-3}_{aqueous}.day^{-1}$.

3.4 Mineralisation - Methanogenesis

The depletion of the methanogenic substrate and methanogen growth are described by Monod kinetics, hence for MB accumulation (r_j),

$$r_j = \frac{k_0 c}{(k_{MC} + c)} m \tag{14}$$

where k_0 is the maximum specific growth rate, k_{MC} is the half saturation constant and m is the MB concentration. The rate of VFA depletion, r_h, is directly related to MB accumulation through a cell/substrate yield coefficient, Y,

$$r_h = \frac{r_j}{Y} \tag{15}$$

The MB decay r_k is given by,

$$r_k = k_2 m \tag{16}$$

where k_2 is the methanogen death rate.

Estimates for the methanogenic parameters were originally sought from a literature review but there was little data relating to MSW, and that which was available covered a range of values (see Table 3). These data provided the starting point for methanogenesis parameter selection and were subsequently refined following a parametric sensitivity study (McDougall & Philp, 2001).

Table 3: Mineralisation parameters and values

Reference	K_0	K_{MC}	Y	k_2
Straub & Lynch (1982) Model waste	0.03 day^{-1}	5000 mg/L	0.04	0.01 day^{-1}
Lee & Donaldson (1985) Cellulose	0.5 day^{-1}	4200 mg/L	0.75	0.02 day^{-1}
Viturtia (1995) Pig manure	0.57 day^{-1}	3280 mg/L	0.19	
El-Fadel (1996) Various	0.25 day^{-1}	500 mg/L	0.06	0.03 day^{-1}

3.5 Transport, growth and decay - governing equations

The combined transport, growth and decay of VFA and methanogenic biomass in the biodegradation model are described by,

$$D_c \frac{\partial^2 c}{\partial x^2} + D_c \frac{\partial^2 c}{\partial z^2} - \frac{q_x}{\theta} \frac{\partial c}{\partial x} - \frac{q_z}{\theta} \frac{\partial c}{\partial z} + [r_g - r_h] = \frac{\partial c}{\partial t} \tag{17a}$$

where D_c is the VFA diffusion coefficient and q is the advective flux; and,

$$D_m \frac{\partial^2 m}{\partial x^2} + D_m \frac{\partial^2 m}{\partial z^2} - \frac{q_x}{\theta} \frac{\partial m}{\partial x} - \frac{q_z}{\theta} \frac{\partial m}{\partial z} + [r_j - r_k] = \frac{\partial m}{\partial t} \tag{17b}$$

where D_m is the methanogenic biomass diffusion coefficient. To date, no distinction between the values assigned to D_c and D_m has been made, mainly because of the dominance of growth and decay terms over diffusion on local species concentrations.

Expansion of the growth and decay terms reveals the nature of the interdependency between VFA and MB concentrations. Two simultaneous partial differential equations are obtained, which are solved iteratively, by updating system parameters until a consistent solution is obtained. The solid organic fraction remaining, S, is decremented at the end of each time step, as described in the next section.

$$D\frac{\partial^2 c}{\partial x^2} + D\frac{\partial^2 c}{\partial z^2} - \frac{q_x}{\theta}\frac{\partial c}{\partial x} - \frac{q_y}{\theta}\frac{\partial c}{\partial z} + \theta_E b\left[1 - \left(\frac{s_o - s}{s_o}\right)^n\right]\exp(-k_{vfa}(c)) - \frac{k_0 c}{(k_{mm} + c)}\frac{m}{Y} = \frac{\partial c}{\partial t} \tag{18a}$$

$$D\frac{\partial^2 m}{\partial x^2} + D\frac{\partial^2 m}{\partial z^2} - \frac{q_x}{\theta}\frac{\partial m}{\partial x} - \frac{q_y}{\theta}\frac{\partial m}{\partial z} + \frac{k_0 c}{(k_{MC} + c)}m - k_2 m = \frac{\partial m}{\partial t} \tag{18b}$$

3.6   Substrate depletion

Solid organic depletion, which leads directly to the accumulation of VFA in the aqueous phase, is determined for each time step. The substrate depletion calculation is made using the modified enzymatic hydrolysis function, Eq(13), which defines the transfer of cellulolytic matter from solid to aqueous phases. However, calculation of solid matter loss as the corollary of VFA accumulation in the aqueous phase must account for the different volumetric denominations and molar masses in each phase. The details of these adjustments are as follows.

a) Reference volumes – the solid degradable content of a landfill is conveniently reported as mass per unit total volume whereas VFA concentrated in landfill leachate is given as mass per unit aqueous phase volume. The modified enzymatic hydrolysis function is therefore multiplied by the volumetric moisture content to ensure the VFA aqueous phase concentration increase is consistent with the loss of solid mass per unit total volume.

b) The solid organic matter lost is taken to be that of cellulose ($C_6H_{10}O_5$), which has a molecular weight of 162 g/mol, whereas the accumulated VFA is assumed to be principally acetic acid (CH_3COOH), which has a molecular weight of 60 g/mol. Therefore, the accumulation of 60g of acetic acid is a result of the solubilisation of 162g of cellulose. A conversion factor reflecting the relative molar masses is therefore required to ensure that the correct amount of cellulose is depleted per unit increase of VFA mass concentration.

Thus substrate depletion is calculated using,

$$S^{t+\Delta t} = S^t - \theta.\frac{162}{60}r_g\Delta t \tag{19}$$

where S^t is the solid degradable fraction remaining in time step t and $t+t$ is the next time step.

3.6.1   Moisture consumption

The stoichiometry of the hydrolytic step shows that 1 mole of water is required to hydrolyse 1 mole of cellulose. The corresponding molar masses dictate that the hydrolysis of 162 g of cellulose consumes 18 g of water, hence,

$$dM_{H_2O} = \frac{18}{162}dM_{cellulose} \tag{20}$$

where M is the mass of water and cellulose as subscripted. Since both volume of moisture and the mass of cellulose are reported to total volume bases, Eq 20 can be divided through by total volume, V_T, from which is obtained,

$$d\theta = \frac{18}{162\rho_{H_2O}}dS \tag{21}$$

where ρ_{H2O} is the density of water. Equation 21 defines the reductions in volumetric moisture content due to water being consumed by hydrolysis. These reductions are handled as local Neumann boundary conditions in the hydraulic model and influence the rate of hydrolysis, working through the effective moisture content term.

As already indicated, the direct conversion of cellulose to acetic acid is predicated on the assumption that acidogenesis and acetogenesis are robust and rapid processes. This means that in the early stages of landfill biodegradation, acetic acid will accumulate, at least until methanogenesis is established, whereupon hydrolysis becomes the rate-limiting step. Such behaviour has been described many times (e.g. Vavilin et al, 2003; Ham & Barlaz, 1993).

3.7   Mass balance

Since mineralisation means that solid mass is not conserved during the course of an analysis, it is both illustrative and important to make a quantitative check on the fate of solid cellulolytic matter. An 'element' test has been devised in which hydraulic and mechanical regimes are constrained to a steady or fixed state, i.e. there is no flow and no deformation. The biodegradation model is then run for a period of 677 days to allow the fate of solid cellulolytic matter to be observed. The environment is conducive to mineralisation, i.e. adequate moisture is available.
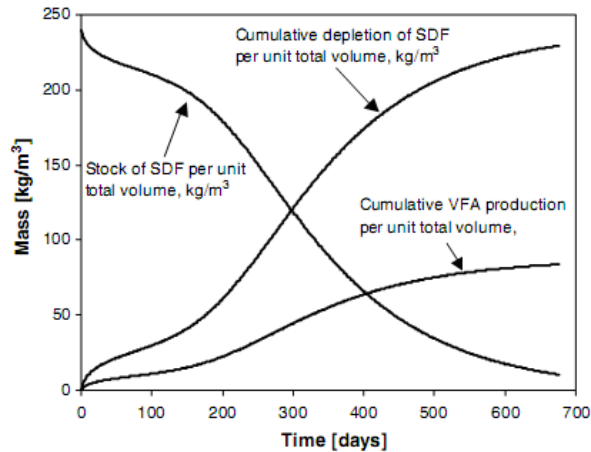
Figure 6. Stock of solid degradable fraction (SDF) per unit total volume (kg/m3), cumulative depletion of SDF per unit total volume and associated cumulative VFA production per unit total volume. VFA production reflects SDF depletion after adjustment by 60/162, i.e. the relative molecular masses of VFA as acetic acid and cellulose.

Figure 6 shows the stock of solid degradable fraction (initially 239.96 kg/m^3) almost completely mineralised during the course of the simulation and all metabolic processes depicted in Figs. 6 & 7 slowing down as a result. Whilst it is not possible to validate the rate of mineralisation, it is interesting to explore and verify the relationships between the various processes of the metabolic pathway. It may be noted, for example, that the cumulative depletion of solid degradable matter is directly proportional to the amount of VFA produced. By presenting the VFA production curve to a unit total volume base, it is evident that the rate of SDF depletion is 2.7 times greater than the rate of VFA accumulation – exactly the ratio of corresponding molecular masses, i.e. 162 & 60. In Fig. 7, the vigour of the VFA accumulation process is clearly evident by comparison to the MB growth process and the depletion of VFA is directly proportional to MB growth by the specified yield coefficient value of 0.08.
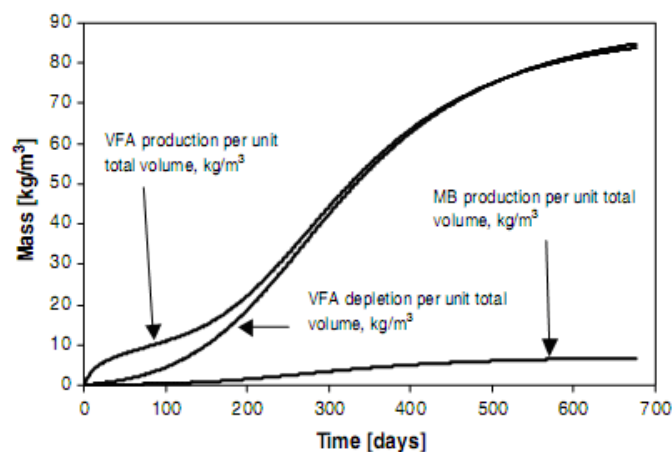
Figure 7. VFA production, VFA depletion and MB accumulation per unit volume.

Of interest is the pattern of substrate depletion, which is neither linear, nor is it obviously a first order decay process. Rather the pattern of substrate depletion is indicative of a combination of factors controlling the decomposition process. A fuller explanation of the simulation of the decomposition process can be found in Section 7.1.4.

4 Mechanical model

The mechanical model combines load, creep and biodegradation-induced effects to predict landfill settlement. Applied loads trigger an immediate elastic or elastoplastic settlement, whereas time-dependent creep and rate-limited biodegradation give rise to long-term settlement. Total settlement can be written as the sum of four main strain components,

$$\varepsilon_i = \varepsilon_i^e + \varepsilon_i^p + \varepsilon_i^c + \varepsilon_i^d \tag{22}$$

where ε_i is strain vector and superscripts *e, p, c* and *d* denote elastic, plastic, time-dependent creep and degradation induced strains respectively.

4.1 Parameter requirements

Parameter requirements are given in the Table 4 below; justification for the selection of hydraulic parameter values is given in the relevant sections.

Table 4. Mechanical model parameters and values for MSW

	Input parameter	*Dimensions*	*Waste*
M1	Elastic stiffness,		0.072
M2	Elastoplastic stiffness,		0.23
M3	Critical state friction constant, $\square$		1.2
M4	Poisson's ratio,		0.35
I4	Initial yield stress	Kpa	30
M5	Creep viscosity		0.0015
M6	Decomposition-induced void change		-0.65
M7	Decomposition hardening	Kpa	2
I5	Dry unit weight (as placed)	kN.m⁻³	5
M8	Inert phase particle weight	kN.m⁻³	17
M9	Degradable phase particle weight	kN.m⁻³	7.3

4.2 Landfill settlement

The five landfill settlement mechanisms: mechanical, ravelling, physico-chemical, biochemical, and interaction, first put forward by Sowers (1973), are now well-known. They are difficult to distinguish individually so are usually interpreted using a three-part temporal classification of initial, primary and secondary settlement stages (Morris & Woods, 1990). However, the interpretation of secondary landfill settlement as a time-dependent process is an expediency which masks the fundamental nature of the biodegradation process. Landfill settlement will therefore be considered here as a combination of separate load, creep and biodegradation settlement processes (McDougall & Pyrah, 2001; 2003)

4.3 Load-induced behaviour - Elasto-plasticity

Although load-induced landfill settlement is usually characterised by a constrained modulus (e.g. Sowers, 1973; Morris & Woods, 1990; Watts & Charles, 1999) evidence of elasto-plastic compression can be obtained from various sources. For example, Kavazanjian et al (1999) measured the compression of reconstituted waste samples in large scale oedometers, shown in Figure 8, from which a clear and consistent difference in elasto-plastic virgin loading and elastic unloading compression coefficients can be observed. Similarly, pressure against cavity strain measurements, obtained by Dixon et al (1999) from self-boring pressuremeter tests in a relatively young waste, presented in Figure 9, show elasto-plastic virgin compression with elastic unload-reload behaviour. Thus the HBM model treats load-induced compression as elasto-plastic using Modified Cam Clay.
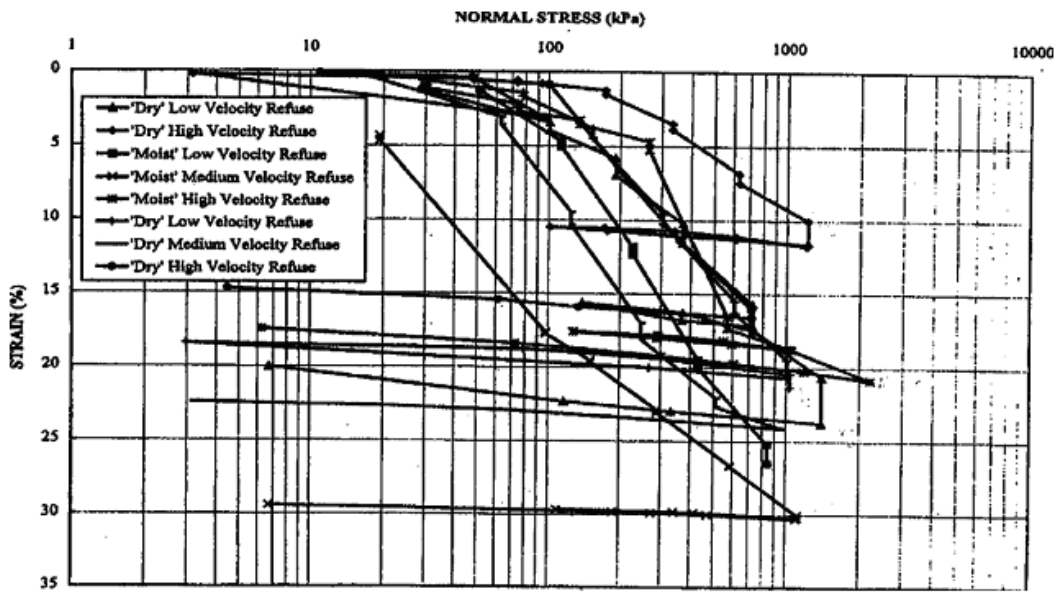
Figure 8. Oedometer test results on waste, reproduced from Kavazanjian et al. [12].
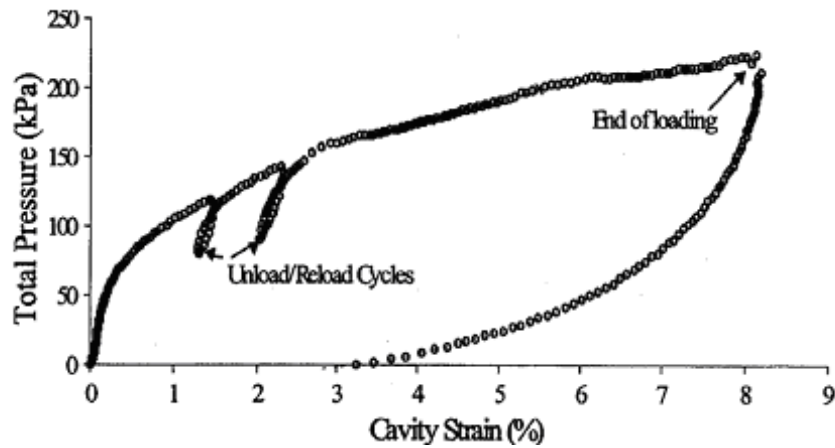

Figure 9. Pressure with cavity strain results obtained from in situ pressuremeter tests, Dixon et al. [7].

4.3.1   Theoretical formulation

The version of Modified Cam Clay in the HBM model is based upon total stress, plane strain stress invariants. A total stress formulation is used because of the unsaturated nature of landfilled waste. Leachate is ordinarily pumped from the base of a site to maintain hydraulic pressure heads of no more than 1 m on the base liner. It is hence assumed that the moisture in the waste mass is not under pressure and therefore it does not contribute to carrying the load. Gas pressure is also not included in the formulation.

$$\sigma_m = \frac{1}{2}\left(\sigma_x + \sigma_z\right) \tag{23a}$$

$$\sigma_d = \sqrt{\left(\sigma_x - \sigma_z\right)^2 + 4\tau_{xz}^2} \tag{23b}$$

where σ_m is the mean stress, σ_d is the deviator stress, σ_x is the horizontal stress, σ_z is the vertical stress and τ_{xz} is the shear stress. Material stiffness is defined in relation to bulk and shear moduli, K & G respectively,

$$K = \frac{v\sigma_m}{\kappa} \tag{24a}$$

$$G = \frac{3}{2}\frac{(1-2\mu)}{(1+\mu)}K \tag{24b}$$

where v is specific volume, μ is Poisson's ratio and κ is elastic stiffness. Equation 24a is based on the definition of $K = d\sigma_m/d\varepsilon_v$. A stricter formulation, allowing for the influence of the out of plane stress, $\sigma_y = \mu(\sigma_x + \sigma_z)$, would give a bulk modulus $K = 2/3 v\sigma_m(1+\mu)/\kappa$. Such a formulation also has consequences for the yield surface. Here, for ease of implementation, the yield function F and associated flow rule are given by

$$F = M^2\sigma_m^2 - M^2\sigma_m h + \sigma_d^2 = 0 \tag{25}$$

where *M* is the critical state friction constant and *h* is a hardening parameter (equivalent to the yield surface tip stress). Plastic volumetric strain hardening occurs according to the following hardening rule,

$$dh^p = \frac{v h}{(\lambda - \kappa)}d\varepsilon_v^p \tag{26}$$

where dh^p is the increment in yield surface tip stress due to plastic volumetric strain, λ is the elastoplastic stiffness, and $\varepsilon^p{}_v$ is plastic volumetric strain.

4.4 Creep behaviour - Visco-elasto-plasticity

Bjerrum (1967) noted during periods of constant loading, the development of a reserve resistance to additional loading. In other words, creep produces an increase in the effective preconsolidation pressure, thereby creating a stage of relatively stiff, elastic straining before elasto-plastic virgin compression is regained. Landva et al (2000) applied alternating increments of vertical stress and creep loading to 9 year old partly decomposed waste in large (450mm) oedometers. Each of the creep loading stages lasted for a period of one week only so none allowed for much (if any) biodegradation. Their results, reproduced in Figure 10, indicate a form of visco-elasto-plastic settlement behaviour identical to that proposed by Bjerrum for more conventional soils.
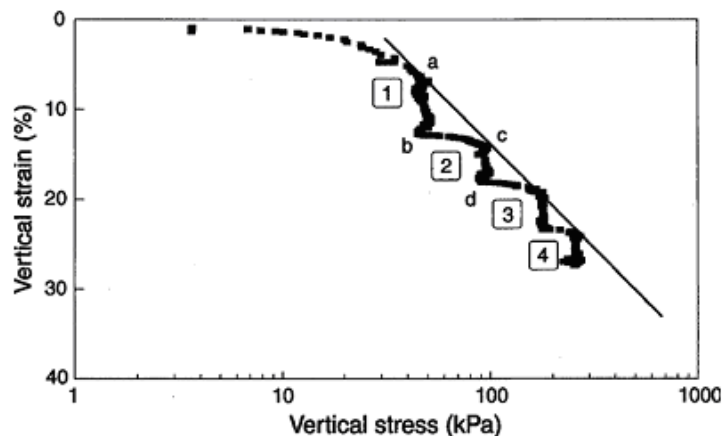

Figure 10. Creep behaviour of waste refuse; reproduced from Landva et al.[13].

Creep behaviour is incorporated within the HBM model using the 'equivalent time' method put forward by Yin & Graham (1989). 'Equivalent time' allows the creep strain rate of an overconsolidated material and its hardening to be related to the normal consolidation line at all stages of loading.

Consider Fig.11, for example. A sample is loaded from an initial stress state at A′ to C. The sample is then left under constant load during which time creep compression moves the sample from C to C′. Continuing creep strain, towards C″, is a function of the time corresponding to the vertical separation of the current volumetric state

with the normal consolidation line; it is this time that is referred to as the equivalent time. In incremental form and for multi-stepped loading,

$$\frac{de}{dt} = -\frac{\chi}{\left(t - t_o + t_{eq} + t_{ref}\right)} \tag{27}$$

where e is void ratio, χ is the creep viscosity coefficient, t is the current time, to is the time at which the current creep stage commences (corresponding to the time at which point C in Fig.11 is reached), teq is the equivalent time and tref is a reference time, in this case tref = 1 day. Yin & Graham (1989) show that where a combination of creep and further loading occurs, as at point D′ in Fig.11, equivalent time can be calculated from,

$$t_{eq} = \left(t_{C'} - t_C - t_{ref}\right)\left(\frac{\sigma_D}{\sigma_C}\right)^{-\left(\frac{\lambda-\kappa}{\chi}\right)} - t_{ref} \tag{28}$$

As creep occurs, there is a change in the effective pre-consolidation pressure. From geometrical considerations, it may be shown that the change in preconsolidation pressure, is given by,

$$\frac{dh^c}{dt} = \sigma_C\left(\frac{\chi}{\lambda-\kappa}\right)\left(\frac{t - t_C + t_{eq} + t_{ref}}{t_{ref}}\right)^{\left(\frac{\chi}{\lambda-\kappa}-1\right)} \tag{29}$$

where dh^c is the increment in yield surface tip stress corresponding to a given period of creep strain from C in Fig.11.
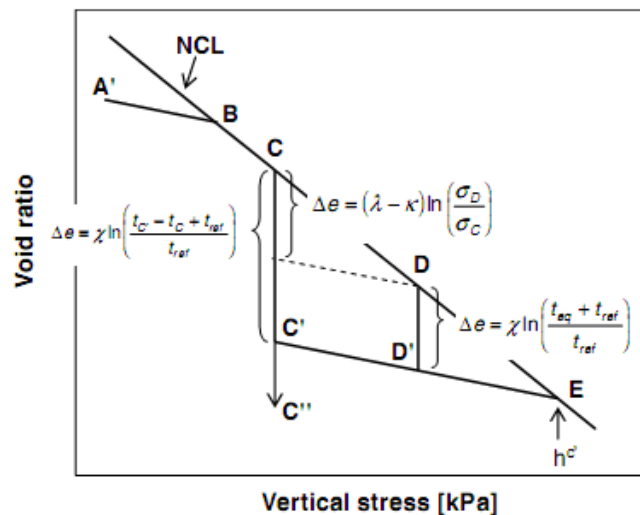

Figure 11. Idealised stepped loading and creep sequence with associated strain response.

4.5 Biodegradation-induced settlement: Bio-plasticity

The progress of biodegradation is known to contribute significantly to the magnitude of secondary settlement in landfills. However, its dependence on biochemical entitities, such as leachate characteristics and microbial activity, and moisture content, means it is difficult to incorporate meaningfully into time-dependent analyses of secondary settlement.

4.5.1 Biodegradation-induced phase changes

Biodegradation-induced settlement is difficult to analyse using conventional soil mechanics volume descriptors because of the changing solid phase mass and volume. For example, the degradation of solid mass may be accompanied by an equivalent (to the solid volume lost) increase in void volume. There is then no change in

overall volume although the void ratio has increased and the waste has become more skeletal. Alternatively, decomposition may be accompanied by contemporaneous particle rearrangement resulting in overall settlement In this case the associated change in void ratio is a function of changes in both void and solid phase volumes; it may even remain unchanged despite overall settlement. McDougall & Pyrah (2004) proposed a constitutive relationship between decomposition of solid degradable fraction, i.e. a change in solid phase volume V_S, and the induced change in void volume V_V at constant stress, of the form,

$$dV_V = \Lambda \, dV_S \tag{30}$$

where Λ is the decomposition (or degradation)-induced void change parameter. Table 5 summarises changes in volumetric state variables and likely mechanical consequences associated with key values of Λ.

Table 5: Decomposition induced void change parameter - reference values and associated phase composition changes where dVs < 0.

$\boldsymbol{\Lambda}$	Void ratio	Overall volume	Phase composition and its expected strength
-1	Maximum increase	No change	Much looser & possibly weaker
0	Increase	Reduction	Looser & possibly weaker
e (= void ratio)	No change	Large reduction	No change
>e	Decrease	Maximum reduction	More compact & possibly stronger

There are two ways in which Λ can be used to interpret biodegradation-induced settlement. Firstly, Λ can be used to quantify the overall impact of decomposition on waste settlement, i.e. to predict the long-term volumetric state for a given combination of material composition and environmental control parameters. On the other hand Λ can be used incrementally to distinguish periods of zero overall volume change from periods of contemporaneous settlement, even accelerated settlement. In either case, the form of Λ can be calculated from compression tests in which the progress of decomposition is known. There is, however, little such data available. Later in this paper, the first quantification of Λ obtained from a large-scale long-term laboratory test on waste refuse (Olivier & Gourc, 2007) is reported. A value of $\Lambda \approx -0.60$ was obtained. Other tests performed by the Author on sand-gypsum and sand-halide mixes, from which the soluble particles were gradually dissolved, indicate that the incremental value of Λ is not constant but changes with the progress of decomposition; more interestingly, the form of the change is not erratic but follows a steady path (unpublished data). If more data on the overall and incremental values of Λ can be obtained, then the HBM model will be closer to operating in a predictive mode. Recall, however, that void phase changes are realised as one-dimensional vertical-only deformations. Quantitatively meaningful simulations of the full HBM formulation can therefore only made on vertical columns.

Whether used as an indicator of overall or incremental behaviour, the use of Λ offers two important advantages over simpler methods. Firstly, biodegradation settlement is not treated as a time-dependent process. Time is communicated through solid organic matter depletion, which is controlled by the biodegradation model. In this way there is a maximum rate of depletion but within that rate, under the influence of moisture deficit/addition or acid accumulation or changing crystallinity, for example, decomposition may slow down, accelerate, or stop completely.

Secondly, this approach allows for the definition of hardening or (more likely) softening with decomposition. Note, however, that the hardening being considered here occurs in response to a change in phase composition, which in turn is induced by degradation of the solid phase and hence controlled by the biodegradation model.

4.5.2   Form and implementation of the biodegradation-hardening rule

It is evident from Table 5 that by combining the void ratio e, with Λ, a simple means of controlling both biodegradation-induced hardening and softening is obtained. When $\Lambda < e$, decomposition leads to an increase in void ratio, whereas when $\Lambda > e$, void ratio decreases. If it is assumed that changes in void ratio impact on waste in the same way as in conventional soils, i.e. by triggering changes in the yield condition then a biodegradation-hardening rule can be defined thus,

$$dh^d = \Omega(e - \Lambda)\frac{dV_S}{V_S} \qquad (31)$$

where dh^d is the increment in yield surface tip stress due to biodegradation, Ω [kPa] is a decomposition hardening multiplier that relates the magnitude of tip stress increments to increments of strain. The qualitative behaviour of the yield condition and its response to the biodegradation-hardening rule is illustrated in Fig. 12. When $\Lambda < e$, decomposition is associated with reductions in yield stress, i.e. the material softens. In contrast, when $\Lambda > e$, solid volume loss produces an increase in yield stress and the material hardens.
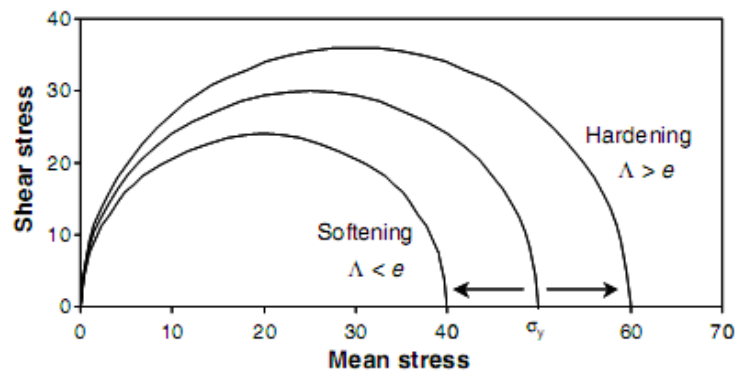
Figure 12. Changes in position of yield surface with decomposition.

In general, the three sources of hardening: load-induced plastic strain, creep and biodegradation are additive, which means that

$$dh = dh^p + dh^c + dh^d \qquad (32)$$

The load-induced plastic components of the strain rate, $\dot{\varepsilon}_i^p$, are calculated from the plastic consistency requirement that $dF = 0$. The creep strain portion of $dh (= dh^c)$ is calculated and applied during period of constant stress during and on completion of a staged filling programme. It is a function of both time and current load as defined by Eq. (29). The assumption of constant stress also applies to the bio-plastic strain rate component, i.e. unloading due to waste weight loss is not currently considered. Bio-plastic strain is proportional to the loss of solid volume, dV_S, as defined by Eq. (31). Recall that dV_S is calculated directly after exit from the biodegradation model and is dependent on combined influence of volatile fatty acids and methanogenic biomass concentrations. Note also that, Ω is a parameter about which very little is known, however, it is possible to perform sensitivity analyses and explore likely value ranges – a value of $\Omega = 2$ has been established in this way.

5   Initial conditions

5.1   Hydraulic variables

An essential part of any simulation is the determination of the initial phase composition, both weights and volumes. Density and moisture content data in conjunction with inert and degradable fractions and phase weights (specific gravities) fix the dry weights, solid and void volumes for hydraulic, biodegradation and mechanical systems. In order to obtain physically consistent suction and moisture content data it is necessary to initialise the hydraulic system by a steady state hydraulic analysis. This procedure is important since physically inconsistent or implausible initial hydraulic conditions will undermine a transient analysis.

5.2   Biodegradation variables

The initial values of biodegradation variables, VFA and MB concentrations, are required inputs. The suggested values have been guided by a parametric sensitivity analysis (McDougall & Philp, 2001).

5.2.1 VFA

An initial VFA concentration of 300 g/m$^3_{aqueous}$ has been adopted, although enzymatic hydrolysis is so vigorous that the initial value has virtually no influence over long-term concentrations and biodegradation. Figure 13, taken from McDougall & Philp (2001), shows long-term VFA concentrations for three different initial values; any distinction between the tests is clearly lost after the first reporting point.
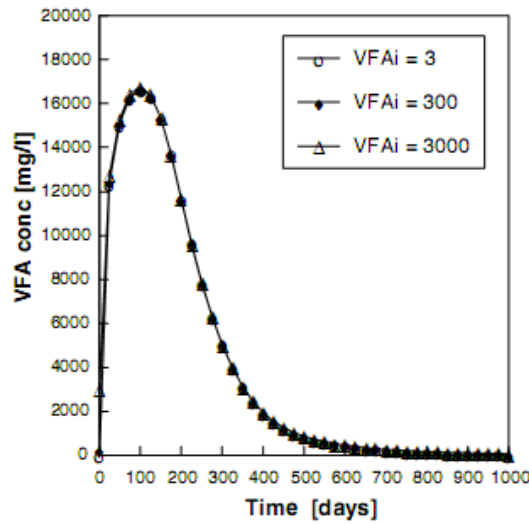

Figure 13. Sensitivity of VFA concentration to initial values (VFAi). All values given in g/m3, taken from McDougall and Philp [17].

5.2.2 Methanogenic biomass

In contrast, the slow accumulation of MB means that initial MB concentrations do influence VFA accumulation and solid matter conversion. Figure 14 shows earlier and lower peak VFA concentrations under the influence of higher initial MB concentrations together with the associated SDF depletion curves. It would appear that high MB concentrations are not essential for SDF depletion, but when present as an initial stock, they do accelerate the onset of methanogenesis.
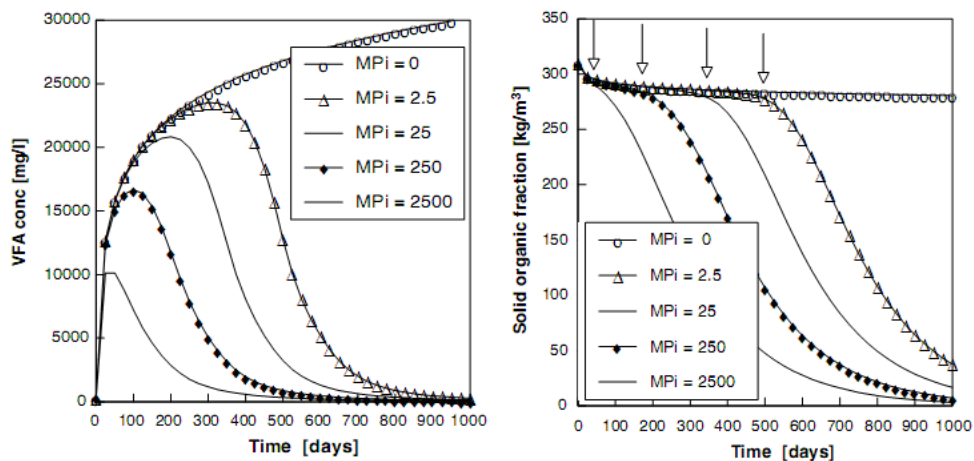

Figure 14. Sensitivity of MB concentration (left) and SDF (right) to initial values (MBi). All values given in g/m3, taken from McDougall and Philp [17].

5.3   Initial stress state

Since the mechanical model is formulated in terms of total stresses there is no need to establish a consistent effective stress regime. Waste, however, being highly compressible requires a plausible variation of density with depth. This is achieved by simulation of the filling phase. As-received waste properties, in conjunction with other mechanical parameters, are used to predict the final loaded waste depth.

6   Implementation

The HBM model is implemented using the finite element method, with each system model sharing a common mesh. In this way it is possible to address material and operational features such as complex section geometry, waste heterogeneity and simulation of the filling phase, see Fig. 15.   By controlling element disclosure, biodegradation and moisture transfers both within the waste and by climatic transfer can be controlled to produce a waste profile that on completion of filling possesses a much more realistic hydro-bio-mechanical condition.
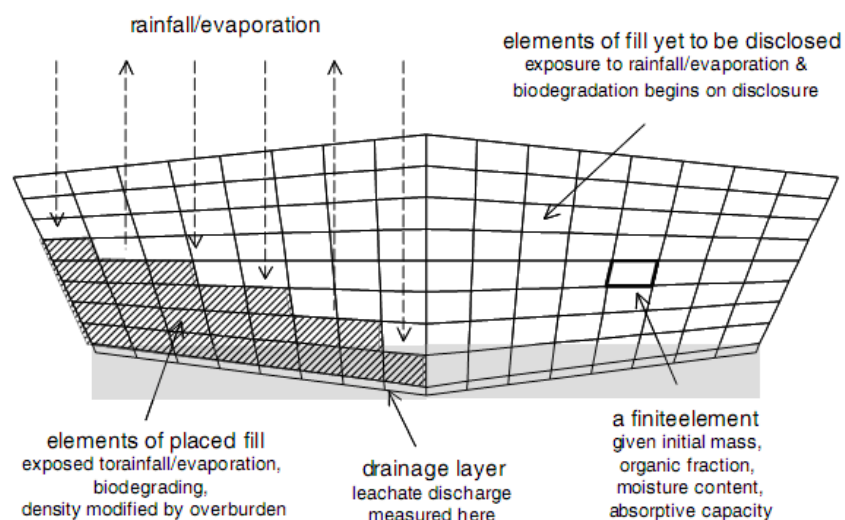

Figure 15. Finite element discretisation illustrating control of filling.

6.1   Link routines

The HBM algorithm guides the solution between system models via link routines (see Fig.1). These link routines contain calculation procedures by which system parameters and other derived data, principally phase composition data, are updated according to the most recent system variable values. A procedure for allocating element-distributed properties to the nodal points is an essential part of each link routine. The procedures executed by each link routine are summarised below.

6.1.1   Hydraulic-biodegradation (HB) link

Updated volumetric moisture content data is available on exit from the hydraulic model. The HB link routine recalculates moisture volumes, gas volumes, element water weight, element total weight, degree of saturation and other phase ratios.

6.1.2   Biodegradation-mechanical (BM) link

The biodegradation model determines the transport, growth and decay of VFA and MB concentrations. The BM link routine uses this information to determine solid mass loss. Solid weight, total weight, gravimetric moisture content and solid volumes are then updated. Induced void phase volume change is determined using   ; all associated phase volumes, ratios, and derived parameters such as unit weights are then updated. The hardening increment dh^d is implemented in this routine.

6.1.3   Mechanical-hydraulic (MH) link

Displacements obtained from the mechanical model are used to update all phase ratios and derived parameters. Residual and saturated moisture contents are updated in preparation for the ensuing hydraulic system solution. Saturated hydraulic conductivity, if density-dependent, is adjusted here.

Although Eq. (22) shows strains as vector quantities, only the elastic and plastic load-induced strains are implemented for two-dimensional displacements. Both creep and biodegradation-induced effects are currently one dimensional, vertical only, displacement fields. Whilst this formulation creates an incongruity within the mechanical model, it simplifies the development and coding of the HBM model and is adequate for column simulations.

6.2   Element types

The HBM model has been developed using four-noded isoparametric quadrilateral elements. Whilst the limitations of lower order elements are recognised for mechanical problems, a number of factors have postponed moves towards the introduction of higher order elements. These factors are:

- Hydraulic. The steady state condition (either hydrostatic or infiltrative) results in a linear variation in the field variable, the hydraulic pressure head. It is unclear at this stage whether transient or mobile effects, such as pronounced variations in pressure head around moving wetting fronts, are better handled with higher order elements or local mesh refinement.

- Biodegradation. The principal output of the biodegradation model is the depletion of the solid organic fraction. The strength of enzymatic hydrolysis and methanogen growth terms in the evolution of VFA and MB concentrations compared with the diffusive and advective transport terms reduces the significance of the order of the element shape function.

- Mechanical. The interpretation of biodegradation in the long-term settlement behaviour of waste and, more importantly, its interpretation within a landfill settlement model, has been of higher priority than the prediction of stress states within the waste mass.

- Large-displacements. Landfill settlement is a large displacement problem yet the mechanical model described here is a small-strain formulation. However, the extent to which the omission of second order terms, four-noded quadrilateral elements, and biodegradation effects each contribute to the total displacement is as yet unverified.

These factors and issues relating to particle compression will be pivotal considerations in any future model developments.

7   Illustrative simulation – lirigm waste compression test

The aim of this simulation is to illustrate the performance and potential of the HBM model by comparison with and reference to a laboratory-scale long-term compression test on MSW undertaken at LIRIGM, University of Grenoble (Olivier & Gourc, 2007). It is the case that the range of parameters required by the HBM model has, to date, not been collected within a single test programme. Validation of only a subset of the required parameters is therefore possible, in this case the mechanical parameters. Hydraulic and biodegradation outputs will also be reported but with only summary comments.

The general strategy of the simulation was firstly to establish the as-placed conditions of the waste; secondly, to determine mechanical parameters by fitting to the initial compression data; finally, and without further adjustment of compression parameters, to explore secondary compression under a range of decomposition-induced void change parameter values.

7.1   LIRIGM, University of Grenoble simulation

A long-term waste loading, settlement and decomposition experiment was performed in the laboratories at LIRIGM, University of Grenoble. The experiment was performed using a cell of approximately 1 m^2 plan area filled with MSW to a depth of 0.845 m. The waste was first incrementally loaded to a total vertical stress of 130 kPa then left under this load for a period of 677 days, during which time leachate discharged from the bottom of the cell was collected and periodically recirculated at the upper surface. A fuller description of the test procedure and outputs can be found in Olivier & Gourc (2007) and Olivier et al (2003); more information on the simulation can be found in McDougall & Hay (2005).

7.2 Initial loading

After establishing initial conditions, the HBM model was run to simulate the first four days of the experiment. Displacements of the surface elevation predicted by the HBM model and experimental data are shown in Fig.16, from which excellent agreement can be noted. It is important to reiterate that these results are obtained by manipulating the mechanical model input parameters to obtain the best fit over the initial compression phase. However, of particular importance was the unload/reload cycle that occurred at about 70 hours into the test. The associated displacement imposes a more stringent control on the combination of acceptable mechanical parameters. This unload/reload loop was produced solely by the removal and subsequent restitution of vertical load, there was no in-run manipulation of compression parameter values**.**
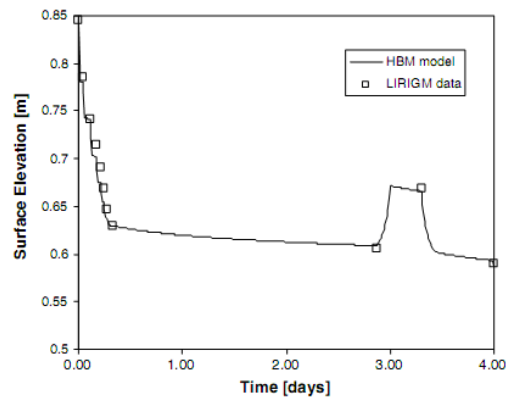

Figure 16. Predicted and measured load/creep behaviour during first 4 days of the experiment.

7.3 Long-term compression

This part of the simulation captures secondary settlement over the entire 677 days of the experiment. No further changes in mechanical compressibility or viscosity parameters were made. The biodegradation model parameters given in Table 2 were used.

The results presented in Fig.17 compare measured and predicted long-term settlement for selected values of Λ, actually 0.0, -0.2, -0.4. Best fit occurs with $\Lambda \approx -0.2$, which corresponds to a loosening of the waste with decomposition. The acceleration in settlement that occurs about 200 days is particularly interesting as it coincides with a period of intensive recirculation, which is shown in Fig. 18, and reveals the influence of increasing moisture content on decomposition. It should be noted that biodegradation-induced settlement is controlled by both the progress of decomposition and the value of the decomposition-induced void change parameter. Thus validation of the value selected for Λ can only be made when the degree of decomposition at the end of the test is determined. According to Olivier & Gourc (2007), 37.1% of the degradable fraction has been lost; in the HBM model the corresponding figure is 39.7%.
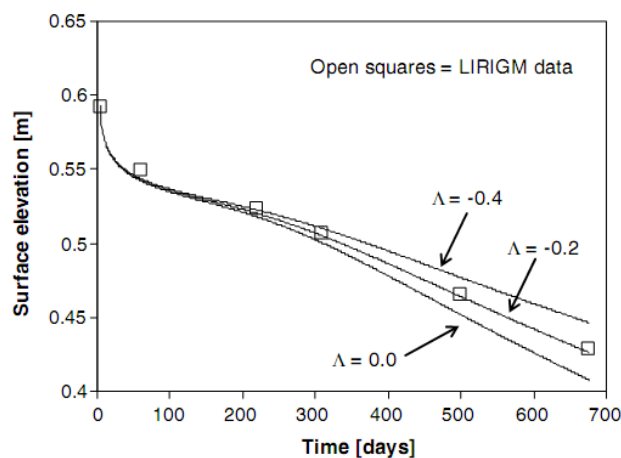

Figure.17. Predicted and measured secondary settlement behaviour during
the experiment for selected values of decomposition-induced void change parameter, Λ.
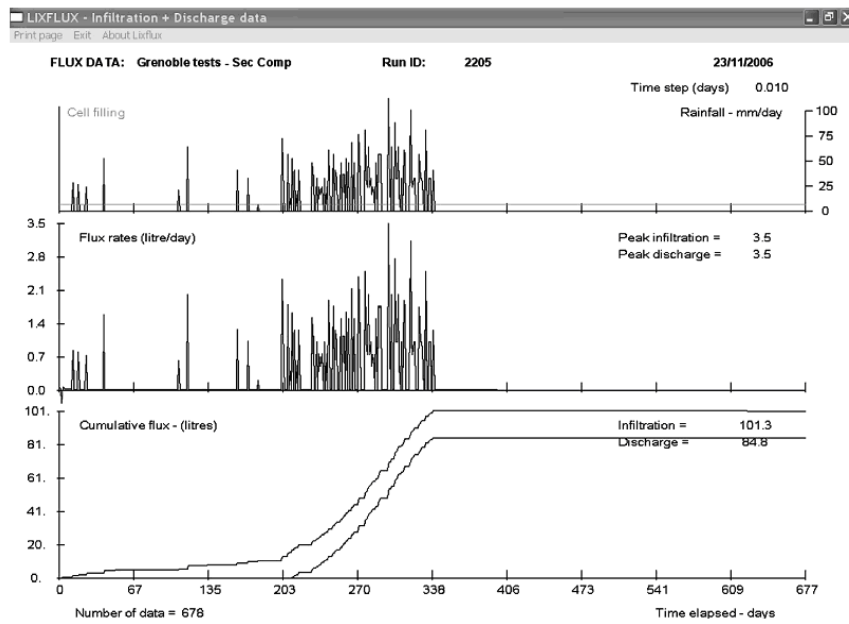
Figure 18. Screen shot from hydraulic graphics routine showing infiltration (as rainfall in mm/day) and predicted fluxes, both rates (l/day) and cumulative amounts (l).

7.4   Other data outputs

To illustrate the capability and fundamental basis of the HBM model a selection of outputs from the hydraulic and biodegradation models is now presented.

VFA & MB concentrations, substrate depletion and gas production

It is instructive to consider the VFA and MB concentration data presented in Figs. 19 & 20, alongside Figs 6 & 7, which show cumulative SDF, VFA and MB production and depletion. It may be seen from Fig.19 that initial VFA accumulation is vigorous, rising to a peak.   Product inhibition, i.e. acid production, triggered by high VFA concentrations, then limits further hydrolysis. There follows a lag phase during which time MB concentrations slowly increase; during these early stages methanogenesis is the rate-limiting step.   As methanogenesis is established, excess VFA is digested so product inhibition is reduced and hydrolysis re-commences.   An equilibrium is then reached in which VFA concentrations are low and SDF depletion rates are high. Continuing rates of conversion are controlled by the availability and digestibility of the remaining degradable fraction. Hydrolysis of the solid organic fraction then becomes the rate-limiting step.   Anaerobic digestion of cellulolytic matter in this form has been described by Barlaz et al (1989) and others.
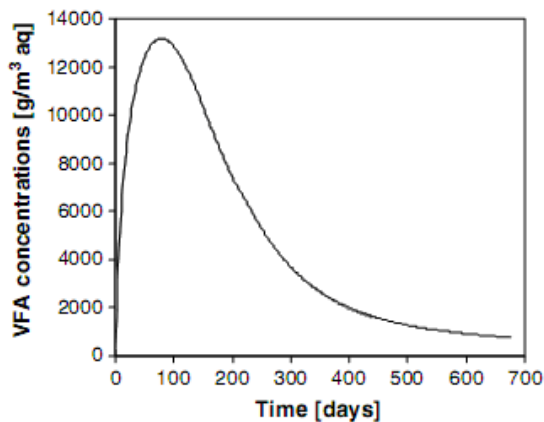

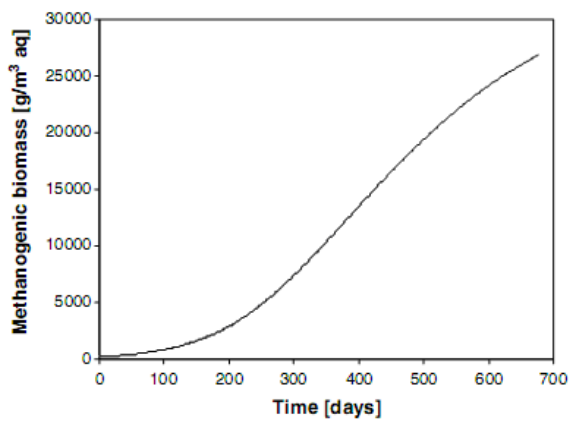Figure 19. Predicted VFA concentration $g/m^3_{aqueous}$ data during LIRIGM test cell.

Figure 20. Predicted MB concentration $g/m^3_{aqueous}$ data during LIRIGM test cell.

Given that the above is a plausible description of landfill biodegradation, it is interesting to note again the form of the substrate depletion curve. It has already been remarked that this is neither linear nor easily fits a first order decay function, yet the latter function is commonly used to define degradation in landfilled waste and associated gas production curves (Thornloe et al., 1999). In fact, crude estimates of landfill gas production can be made by reference to the substrate depletion curves. Such a curve is shown in Fig. 21, which shows quite clearly an initial lag phase during which gas production rises to a peak rate. Simple first-order decay functions have no lag phase but decline gradually from a high initial high value. Thus, the HBM model captures a more fundamental interpretation of waste decomposition processes.
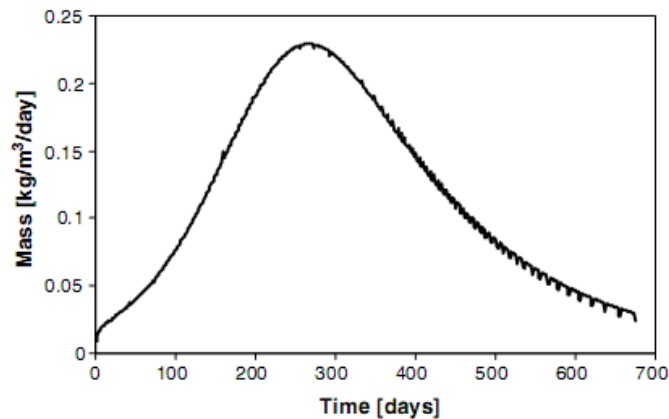

Figure 21. Predicted gas production curve, kg $C_{equivalen}/m^3/day$ based on substrate depletion.

8   Concluding remarks

This paper has presented the formulation of a hydro-bio-mechanical model for the analysis and prediction of landfill processes, with particular emphasis on long-term settlement. The HBM model is innovative in the way that it is not constrained to time-dependent functional descriptions of decomposition nor secondary settlement in landfilled waste. In the HBM model, time is used to control the progress of decomposition, in conjunction with other influential factors such as moisture content, VFA accumulation and methanogen population. The HBM model then translates decomposition into changes in void volume and associated softening (or hardening) using a constitutive relationship between decomposition and induced change in void volume. In this way, biodegradation and settlement is conveniently disengaged so highly empirical approaches are avoided and the best modelling techniques in each can be used. One of the achievements of the approach is its capability to capture accelerated settlement rates due to degradation, which in turn has been stimulated by moisture addition.

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